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We will write

$$\lim_{x\to a} f(x) = L$$

$$\varepsilon > 0 \ \ \delta > 0$$

$$\left|f(x) - L\right| < \varepsilon \text{ if } 0 < \left|x - a\right| < \delta$$

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Also (HW). Read 2.5 - Continuity.

This is an important milestone.

In some respects, this was the impetus (driving force) to pin down a precise definition of limit.

You will also be expected to know def^{n} of continuity

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2.3) 32)

$$\lim_{x\to+\infty} \frac{\sqrt{x^2-3x}-x}{1} \cdot \frac{\sqrt{x^2-3x}+x}{\sqrt{x^2-3x}+x}$$

indeterminate $\infty - \infty$

$(a-b)(a+b) = a^2-b^2$

$$= \lim_{x\to+\infty} \frac{(x^2-3x)-x^2}{\sqrt{x^2-3x}+x}$$

$$= \lim_{x\to+\infty} \frac{-3x}{\sqrt{x^2-3x}+x} = \lim_{x\to+\infty} \frac{x(-3)}{\sqrt{x^2}\sqrt{1-\frac{3}{x}}+x}$$

indeterminate $\frac{\infty}{\infty}$

$\sqrt{x^2-3x} = \sqrt{x^2}\sqrt{\left(1-\frac{3}{x}\right)}$

$\sqrt{x^2} = |x|$

$\sqrt{ab} = \sqrt{a}\sqrt{b}$

$\sqrt{(-2)(-3)}$

$$= \lim_{x\to\infty} \frac{x(-3)}{x\left(\sqrt{1-\frac{3}{x}}+1\right)}$$

because we care only about positive x's,
$\sqrt{x^2} = |x| = x$

$$= \lim_{x\to\infty} \frac{-3}{\sqrt{1-\frac{3}{x}}+1} = \boxed{-\frac{3}{2}}$$

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2.3) 32) $\lim_{x\to+\infty} \dfrac{\sqrt{x^2-3x}-x}{1}\cdot\dfrac{\sqrt{x^2-3x}+x}{\sqrt{x^2-3x}+x}$

indeterminate $\infty-\infty$

$(a-b)(a+b) = a^2-b^2$

$= \lim_{x\to+\infty} \dfrac{(x^2-3x)-x^2}{\sqrt{x^2-3x}+x}$

$= \lim_{x\to+\infty} \dfrac{-3x}{\sqrt{x^2-3x}+x} = \lim_{x\to+\infty} \dfrac{x(-3)}{\sqrt{x^2}\sqrt{1-\frac{3}{x}}+x}$

indeterminate $\frac{\infty}{\infty}$

$\sqrt{x^2-3x} = \sqrt{x^2\left(1-\frac{3}{x}\right)}$

$\sqrt{x^2} = |x|$

$\sqrt{ab} = \sqrt{a}\sqrt{b}$

$\sqrt{(-2)(-3)}$

$= \lim_{x\to\infty} \dfrac{x(-3)}{-x\left(\sqrt{1-\frac{3}{x}}+1\right)}$

because we care only about ~~positive~~ x's, $\sqrt{x^2} = |x| = -x$

$= \lim_{x\to\infty} \dfrac{+3}{\sqrt{1-\frac{3}{x}}+1} = \boxed{+\frac{3}{2}}$

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18) $\lim_{x\to\infty} \sqrt[3]{\dfrac{3x^7-4x^5}{2x^7+1}}$

$= \lim_{x\to\infty} \dfrac{\sqrt[3]{x^7}\sqrt[3]{3-\frac{4}{x^2}}}{\sqrt[3]{x^7}\sqrt[3]{2+\frac{1}{x^7}}}$

To learn math, you must invent it yourself.

$= \sqrt[3]{\lim_{x\to\infty} \dfrac{x^7(3-\frac{4}{x^2})}{x^7(2+\frac{1}{x^7})}}$

$= \sqrt[3]{\lim_{x\to\infty} \dfrac{3-\frac{4}{x^2}}{2+\frac{1}{x^7}}}$

$= \sqrt[3]{\dfrac{3-0}{2+0}}$

$= \sqrt[3]{\dfrac{3}{2}}$

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34) $$\lim_{x\to\infty} \frac{\left(\sqrt{x^2+ax}-\sqrt{x^2+bx}\right)}{1} \cdot \frac{\sqrt{x^2+ax}+\sqrt{x^2+bx}}{\sqrt{x^2+ax}+\sqrt{x^2+bx}}$$

$$= \lim_{x\to\infty} \frac{(x^2+ax)-(x^2+bx)}{\sqrt{x^2+ax}+\sqrt{x^2+bx}}$$

$$= \lim_{x\to\infty} \frac{(a-b)x}{\sqrt{x^2}\left(\sqrt{1+\frac{a}{x}}+\sqrt{1+\frac{b}{x}}\right)}$$

$$= \lim_{x\to\infty} \frac{a-b}{\sqrt{1+\frac{a}{x}}+\sqrt{1+\frac{b}{x}}} = \frac{a-b}{2}$$

HI! \(^_^)/